



C-14-CHOT/M/RAC-102

4050

BOARD DIPLOMA EXAMINATION, (C-14)

APRIL/MAY—2015

DME—FIRST YEAR EXAMINATION

ENGINEERING MATHEMATICS—I

Time : 3 hours]

[Total Marks : 80

PART—A

3×10=30

Instructions : (1) Answer **all** questions.

(2) Each question carries **three** marks.

1. Resolve $\frac{3x}{(x-2)(x-1)}$ into partial fractions.

2. If $A = \begin{pmatrix} 2 & 5 & 3 \\ 7 & 6 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 2 & 7 \\ 3 & 5 & 4 \end{pmatrix}$, verify that $(A+B)^T = A^T + B^T$.

3. If ω is a cube root of unity, then prove that $\begin{vmatrix} 1 & & 2 \\ & \omega & 1 \\ 2 & 1 & \end{vmatrix} = 0$.

4. If $\tan A = \frac{5}{6}$ and $\tan B = \frac{1}{11}$, then show that $\tan(A+B) = 1$.

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 5. Prove that $\sin A \sin(60^\circ - A) \sin(60^\circ + A) = \frac{1}{4} \sin 3A$.
6. Find the additive and multiplicative inverse of the complex number $4 - 3i$.
7. Find the distance between the parallel lines $3x - 2y - 9 = 0$ and $3x - 2y - 12 = 0$.
8. Find the equation of the circle whose centre is $(1, 2)$ and radius is 5.
9. Evaluate $\lim_{n \rightarrow \infty} \frac{1^2 + 2^2 + 3^2 + \dots + n^2}{n^3}$.
10. If $x = at^2$, $y = 2$, then find $\frac{dy}{dx}$.

PART—B

10×5=50

- Instructions :** (1) Answer *any five* questions.
 (2) Each question carries **ten** marks.

11. (a) Find the adjoint of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 3 \\ 6 & 2 & 1 \end{bmatrix}$.

(b) Solve the equations $x + 2y + 3z = 6$, $2x + 4y + z = 7$ and $3x + 2y + 9z = 14$ by using Cramer's rule.

12. (a) If $A + B + C = 180^\circ$, then prove that $\sin 2A + \sin 2B + \sin 2C = 4 \cos A \cos B \sin C$

(b) If $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \pi$, then prove that $x + y + z = xyz$.

13. ^{*} (a) Solve $\sqrt{3} \cos \theta = \sin \theta$.
 (b) Solve the triangle ABC with $a = 1$, $b = \sqrt{3}$, $c = 2$.
14. (a) Find the centre, vertices, eccentricity, foci, LLR and equations of the directrices of the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$.
 (b) Find the equation of the rectangular hyperbola whose focus is the point $(1, -5)$ and directrix is $x - y - 3 = 0$.
15. (a) If $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \dots}}}$, then find $\frac{dy}{dx}$.
 (b) If $y = ae^x + be^{-x}$, then show that $\frac{d^2y}{dx^2} - y = 0$.
16. (a) Find $\frac{dy}{dx}$, if $x^3 + y^3 - 3axy = 0$.
 (b) If $u = \log(x + y + z)$, then prove that $x \frac{u}{x} + y \frac{u}{y} + z \frac{u}{z} = 1$.
17. (a) Find the lengths of the tangent, normal, subtangent and subnormal for the curve $y^2 = 4x$ at $(1, 2)$.
 (b) A light is hung 8 m directly above a straight horizontal floor. A man 2 m tall is walking away from the lamp at the rate of 5.4 m/min. Find the rate at which his shadow is lengthening.
18. (a) Find the maximum and minimum values of $2x^3 - 9x^2 + 12x - 15$.
 (b) The radius of a spherical balloon is increased by 1%. Find the approximate percentage increase in its surface area.
